

Factor completely over the set of rational numbers (**no complex factoring**).

- 1) $8x^3 + 8x^2 + 7x + 7$
 $(8x^3 + 8x^2) + (7x + 7)$
 $8x^2(x + 1) + 7(x + 1)$
 $(x + 1)(8x^2 + 7)$
- 2) $t^2 - t - 7$
 $ac = -7, \text{no #'s work}$
 Prime, cannot be factored with the integers
- 3) $6m^2 + 24m$
 $6m(m + 4)$
- 4) $5w^2 - 10w - 15$
 $5(w^2 - 2w - 3)$
 $ac = 1(-3) = -3 \text{ and } 1(-3) = -3$
 $\text{and } 1 + -3 = -2$
 $5[(w^2 + 1w) + (-3w - 3)]$
 $5[w(w + 1) - 3(w + 1)]$
 $5(w - 3)(w + 1)$
- 5) $x^2 - 49$
 $(x + 7)(x - 7)$
- 6) $125 + 64x^3$
 $5^3 + (4x)^3$
 $(5 + 4x)(5^2 - 5(4x) + (4x)^2)$
 $(5 + 4x)(25 - 20x + 16x^2)$
- 7) $w^2 - w - 6$
 $ac = 1(-6) = -6 \text{ and } 2(-3) = -6$
 $\text{and } 2 + -3 = -1$
 $(w^2 + 2w) + (-3w - 6)$
 $w(w + 2) - 3(w + 2)$
 $(w - 3)(w + 2)$
- 8) $-15x^3y^2 - 10xy^2$
 $-5xy^2(3x^2 - 2)$
- 9) $6n^2 - 23n + 20$
 $ac = 6(20) = 120 \text{ and } 8(15) = 120 \text{ and}$
 $-8 + -15 = -23$
 $(6n^2 - 8n) + (-15n + 20)$
 $2n(3n - 4) - 5(3n - 4)$
 $(3n - 4)(2n - 5)$
- 10) $25y^2 - 64$
 $(5y + 8)(5y - 8)$
- 11) $2b^2 - 1b + 2$
 $ac = 4, \text{no #'s work}$
 Prime, cannot be factored with the integers
- 12) $8k^2 + 11k - 10$
 $ac = 8(10) = -80 \text{ and } -5(16) = -80 \text{ and}$
 $-5 + 16 = 11$
 $(8k^2 - 5k) + (16k - 10)$
 $k(8k - 5) + 2(8k - 5)$
 $(k + 2)(8k - 5)$
- 13) $-2x^2 + 7x + 15$
 $-1(2x^2 - 7x - 15)$
 $ac = (2)(15) = -30,$
 $\text{and } (-10)(3) = -30$
 $-1[(2x^2 - 10x) + (3x - 15)]$
 $-1[2x(x - 5) + 3(x - 5)]$
 $-1(2x + 3)(x - 5)$
- 14) $6xy - 2y - 3x + 1$
 $2y(3x - 1) - 1(3x - 1)$
 $(2y - 1)(3x - 1)$

$$\begin{aligned}
 15) \quad & 54 - 16x^3 \\
 & 2(27 - 8x^3) \\
 & 2(3^3 - (2x)^3) \\
 & 2(3 - 2x)(3^2 + 3(2x) + (2x)^2) \\
 & 2(3 - 2x)(9 + 6x + 4x^2)
 \end{aligned}$$

$$\begin{aligned}
 17) \quad & y^4 + y^2 - 20 \\
 & (y^2)^2 + (y^2) - 20 \\
 & (y^2 + 5)(y^2 - 4) \\
 & (y^2 + 5)(y + 2)(y - 2)
 \end{aligned}$$

$$\begin{aligned}
 19) \quad & x^4 - 8x^2 - 9 \\
 & (x^2)^2 - 8(x^2) - 9 \\
 & (x^2 + 1)(x^2 - 9) \\
 & (x^2 + 1)(x + 3)(x - 3)
 \end{aligned}$$

$$\begin{aligned}
 16) \quad & 4y^2 + 28y + 48 \\
 & 4(y^2 + 7y + 12) \\
 & ac = (1)(12) = 12, \\
 & \text{and } (4)(3) = 12 \\
 & 4[(y^2 + 4y) + (3y + 12)] \\
 & 4[y(y + 4) + 3(y + 4)] \\
 & 4(y + 3)(y + 4)
 \end{aligned}$$

$$\begin{aligned}
 18) \quad & x^5 - x^3 + 8x^2 - 8 \\
 & x^3(x^2 - 1) + 8(x^2 - 1) \\
 & (x^3 + 8)(x^2 - 1) \\
 & (x + 2)(x^2 - 2x + 4)(x + 1)(x - 1)
 \end{aligned}$$

Factor each quadratic expression completely over the set of complex numbers.

$$\begin{aligned}
 20) \quad & 12m^2 + 12 \\
 & 12(m^2 + 1) \\
 & 12(m^2 - (-1)) \\
 & 12(m + i)(m - i)
 \end{aligned}$$

$$\begin{aligned}
 22) \quad & x^2 + 4x + 5 \\
 & x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{-4}}{2} \\
 & x = \frac{-4 \pm 2i}{2} = -2 \pm i \\
 & (x + 2 + i)(x + 2 - i)
 \end{aligned}$$

$$\begin{aligned}
 21) \quad & -4x^2 - 8 \\
 & -4(x^2 + 2) \\
 & -4(x^2 - (-2)) \\
 & -4(x + i\sqrt{2})(x - i\sqrt{2})
 \end{aligned}$$

$$\begin{aligned}
 23) \quad & x^2 - 3x + 6 \\
 & x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(6)}}{2(1)} = \frac{3 \pm \sqrt{-15}}{2} \\
 & x = \frac{3 \pm i\sqrt{15}}{2} \\
 & 4\left(x - \frac{3 + i\sqrt{15}}{2}\right)\left(x - \frac{3 - i\sqrt{15}}{2}\right)
 \end{aligned}$$

